Microcausality, Sorkin's Paradox, and (Un)measurable Observables

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Abstract **Microcausality**, one of the general principles of relativistic quantum field theory, says that observables localized in causally disjoint regions of spacetime should commute with each other. This principle is meant to ensure that measurements performed in causally disjoint regions cannot influence each other, but **Sorkin's paradox** shows that microcausality by itself is not enough. This article reviews Sorkin's paradox and uses it as a reminder that some of the things we formally designate as "observables" may not actually be measurable by any physical process.

Contents

1 Introduction

Microcausality is one of the general principles of relativistic quantum field theory (QFT).[1] It says that if two regions of spacetime, say A and C, are not causally connected to each other (cannot be connected to each other by any causal worldline),[2] then all observables localized in A should commute with all observables localized in C. This is meant to ensure that the occurrence of a measurement in A cannot influence the distribution of outcomes of a measurement in C, or conversely. Complications arise, though, when we consider the effect of measurements in a third region, say B, that intersects the causal future of A and the causal past of C.[3] If we naïvely assumed that every operator formally designated as an *observable* can actually be measured, then we could choose an observable in B whose measurement would seem to allow the distribution of outcomes of a measurement in C to be influenced by the occurrence of a measurement in A, the very phenomenon that the microcausality principle is meant to exclude. This is called **Sorkin's paradox**. Section 8 shows how to formulate it mathematically.

Sorkin's paradox is a valuable reminder that measurement is a physical process that requires physical resources (section 3), so some of the operators designated as *observables* in a given model may not actually be measurable. Sections 9-10 explain why a proper analysis of Sorkin's paradox should account for that constraint.

[1] Article 21916 introduces some of these general principles. Section **??** will review microcausality.

[2] Article 48968

[3] Section 7 illustrates the geometry.

2 Notation and conventions

This article uses the same conventions and perspectives as articles 03431 and 21916. Observables are represented by linear operators on a Hilbert space. A **state** is represented as a normalized positive linear functional from the algebra of operators to the field of complex numbers. The complex number it assigns to an operator X is denoted $\rho(X)$. The functional corresponding to a state-vector $|\psi\rangle$ is[4]

$$\rho(X) \equiv \frac{\langle\psi|X|\psi\rangle}{\langle\psi|\psi\rangle}.$$

An **observable** A will be represented by a set $A_1, A_2, ...$ of mutually orthogonal projection operators that add up to the identity operator I:

$$A_k^2 = I \qquad A_k^\dagger = A_k \qquad \sum_k A_k = I \qquad A_j A_k = 0 \text{ for } j \neq k. \tag{1}$$

Each of those projection operators represents one of the possible outcomes when that observable is measured. The heisenberg picture will be used, so observables are time-dependent and states are not. **Born's rule** says that if an observable A is (sharply) measured in the state $\rho(\cdots)$, then the probability of obtaining the kth possible outcome is $\rho(A_k)$. If the outcome is known, then the state that should be used as the input to the next application of Born's rule is

$$\rho(\cdots|A_k) \equiv \frac{\rho(A_k \cdots A_k)}{\rho(A_k)}. \tag{2}$$

I'll call this the **selective state-update rule**. If the A-measurement occurs but the outcome is unknown or if we choose not to use any knowledge of the outcome, then the state that should be used as the input to the next application of Born's rule is

$$\rho'(\cdots) \equiv \rho(\cdots|A_k)\rho(A_k) = \sum_k \rho(A_k \cdots A_k). \tag{3}$$

I'll call this the **non-selective state-update rule**.

[4] More generally, the functional corresponding to a density matrix $\hat\rho$ is $\rho(X) \equiv \text{trace}(X\hat\rho)$.

3 What is a measurement?

Born's rule and the state-update rules should not be applied arbitrarily. They should only be applied to observables that are actually measured.

How do we know when an observable has been (or will be) measured? Any answer to that question must be ambiguous to some degree, even if it is clear enough for most practical applications. This unavoidable ambiguity is the essence of the *measurement problem*, and we're not going to solve it here.[5] This section has a more humble goal: to offer a conceptual definition of *measurement* that, though ambiguous, is clear enough for practical applications.[6]

A measurement is a physical process. It requires physical resources. It has physical consequences even if we ignore its outcome. Those consequences have a characteristic quality that we can use as the defining property of *measurement*. This article is concerned only with high-quality measurements, not with all of the naturally occurring measurement-like phenomena that are normally included in the subject called **decoherence**, but we should remember that the line is arbitrary, like the line between what we call a *stream* and what we call a *river*. The definition involves more than just the thing being measured. The rest of the system (the measurement apparatus and its surroundings) plays an essential role. A (**high-quality** or **sharp**) **measurement** may be roughly defined as a physical process in which the system is affected in different ways by the different possibilities distinguished by that measurement, and – here's the key – the difference is practically irreversible.[7]

For that definition to be useful, we need to clarify what *practically irreversible* means. Irreversible with respect to what kinds of operations? Empirically, we know that many of the things that we formally designate as *observables* in a model like quantum electrodynamics are not actually measurable in practice because even

[5] Many (probably most) physicists have questioned whether it really is a problem at some point in their career, many (probably most) physicists have changed their minds about it more than once, and at least few of them have given us some now-famous quotes to validate just how confusing it can be.

[6] This section is an abbreviated version of part of article 03431.

[7] Article 03431

the whole galaxy would not have the resources to measure them.[8] *Practically irreversible* means that the whole-system state-vectors corresponding to different outcomes of the measurement[9] are practically orthogonal to each other (in the same sense that two randomly-chosen vectors in a jillion-dimensional inner product space are practically always practically orthogonal to each other) and remain so no matter what sequences of (feasible) measurements may occur the future. In other words: we may apply the non-selective state-update rule (3) whenever doing so would not have any noticeable effect on any predictions about feasible future measurements. This is the essence of the concept of *decoherence*, here applied to high-quality measurements.[10]

Notice the circularity implicit in this definition: to diagnose the occurrence of a measurement, we need to have a way of identifying which future observables could feasibly be measured, which in turn relies on our ability to diagnose the occurrence of a measurement... To break this cycle, we should hypothesize which so-called observables are actually measurable, and then we should check that our hypothesis is self-consistent with respect to the definition in the previous paragraph. I'm not aware of any theorems regarding the existence (or the degree of non-uniqueness) of such self-consistent observable-sets in natural models like quantum electrodynamics. This is unexplored – and maybe unexplorable – territory, so the best we can do for now is to acknowledge our assumptions and press on. This article simply assumes that such an observable-set exists[11] without trying to describe it explicitly and without trying to check its self-consistency.

[8]One example is an observable whose individual outcomes correspond to quantum superpositions of macroscopically distinct locations of the moon.

[9]Example: if the outcome of the measurement is recorded, then the recording process necessarily dissipates heat, leads to an outcome-dependent quantum microstate of the jillions of molecules in the laboratory's atmosphere.

[10]Much of the literature about decoherence uses models with a prescribed distinction between the part of the system being measured and the rest of the system, usually implemented as different tensor factors of the full Hilbert space. No such prescribed distinction exists in natural models like quantum electrodynamics, where everything – the thing being measured, the measurement apparatus, the laboratory – is governed by the same terms in the hamiltonian, and the very existence of such macroscopic objects is encoded entirely in the initial state.

[11]Existence might require limiting the scope of applications. This is certainly true if the Hilbert space has only a finite number of dimensions, no matter how large that number may be (article 03431).

4 Two approaches for handling measurement

We should distinguish between two different ways of handling measurement in quantum theory:

- Section 5 will describe the *natural approach*, which is better in principle but usually too difficult in practice.

- Section 6 will describe the *artificial approach*, which is inferior in principle but much easier in practice.

When an observable is measured, quantum theory does not predict which of the possible outcomes we will experience. The natural and artificial approaches both rely on Born's rule and the selective state-update rule (2) for handling specific outcomes. In contrast, the non-selective state-update rule (3) has a different status in the different approaches. Sections 5-6 review this distinction, which will be important in the analysis of Sorkin's paradox in sections 8-10.

5 The natural approach

The **natural approach** uses a model that treats the whole system as a single self-contained quantum system, including the measurement apparatus and its surroundings as well as the thing being measured. In this approach, the occurrence or non-occurrence of a measurement is determined by the state ρ, which encodes whatever prior information we have about the configuration of the whole system – any prior manipulations of the thing being measured, the layout of the measurement apparatus, the temperature of the ambient air, and so on. By definition, if we use a state ρ in which a sharp A-measurement occurs, then the state automatically satisfies

$$\rho(\Omega^\dagger\Omega) = \sum_k \rho(A_k\Omega^\dagger\Omega A_k) + \epsilon \tag{4}$$

whenever Ω is a product of projection operators representing a sequence of outcomes of measurements localized in spacetime regions outside the causal past of the region where the A-measurement occurs, where the discrepancy $|\epsilon| \lll 1$ is utterly negligible.[12,13] This is a consequence of the verbal definition of *measurement* reviewed the beginning of this section. The condition that ϵ is utterly negligible for all such Ω corresponds to the *practically irreversible* quality emphasized in section 3. Equation (4) holds only for Ωs that can arise from measurements that can actually occur within the model. It does not hold for most operators Ω, it does not hold for sequences of "observables" whose measurement would not be possible within the model, and it does not hold for states in which an A-measurement does not occur.

In this approach, the non-selective state-update rule (3) is superfluous: if we don't want to use (or don't have) any knowledge of the outcome of the A-measurement, then applying (3) doesn't make any noticeable difference because the state already has the property (4) as a consequence of the model's dynamics.

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[12] Article 03431

[13] Analyses quantifying this in simplified models can be found in literature about **decoherence**. Neglecting ϵ is the same as treating states of the environment correlated with different measurement outcomes as exactly orthogonal.

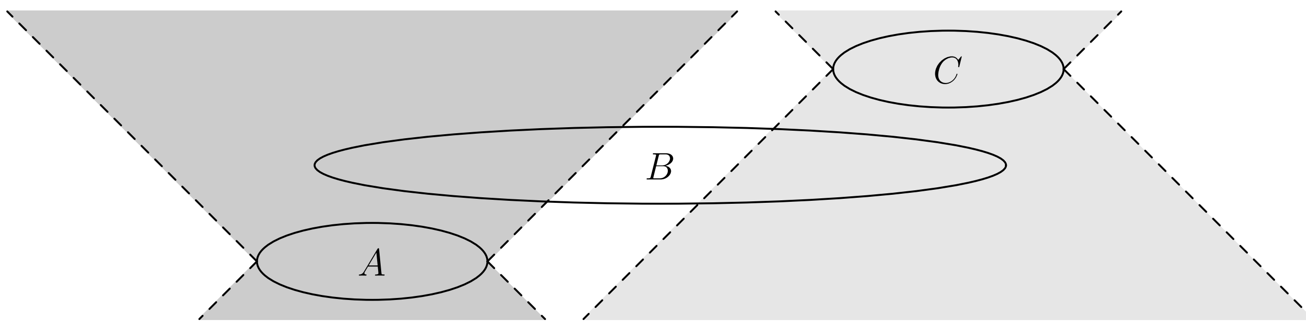
6 The artificial approach

In the **artificial approach**, the non-selective state-update rule (3) is necessary. If we don't know the outcome of the measurement but do know that a given observable was measured, then we should apply (3) manually to account for the measurement's physical consequences. This is a bookkeeping task that compensates for omitting the measurement apparatus and its surroundings from the quantum system being modeled.

Most applications of quantum theory use the artificial approach because it's much easier than the natural approach, but the natural approach is still important conceptually. The paradox reviewed in section 8 is an example of what can go wrong conceptually when the artificial approach is used.

7 The geometric context for Sorkin's paradox

The paradox that will be reviewed in section 8 refers to three spacetime regions A, B, C, where B intersects the causal future of A and the causal past of C, but A and C are not causally connected. An example in two-dimensional spacetime is illustrated here:



The vertical direction is timelike, the future is upward, and the horizontal direction is spacelike. The regions A, B, C are each represented as an ellipse. The shaded areas represent the causal past and future of the regions A and C. Region B overlaps the causal future of A and the causal past of C, but regions A and C are not causally connected: they are outside each other's shaded areas.

8 Sorkin's paradox in the artificial approach

Sorkin's paradox[14] uses three spacetime regions A, B, C that are related to each other as described in section 7. Choose an observable in each region, and use the same letters A, B, C to denote these observables. Each observable is represented by a list of mutually orthogonal projection operators labeled by subscripts, so $A_1, A_2, ...$ are the projection operators representing the A-observable, and similarly for the B and C observables.[15] Microcausality says that the A_ks commute with the C_js, because the regions A and C are mutually spacelike. Using (1), this implies

$$\sum_k \rho(A_k C_j A_k) = \rho(C_j)$$

for any state $\rho(\cdots)$. This says that if the B-measurement did not occur, then the A-measurement would not affect the outcome statistics of the C-measurement.

Now, using what section 6 called the *artificial approach*, suppose that the A- and B-measurements both occur. Then the expectation value of C_j (if not conditioned on specific outcomes of the A- or B-measurements) is

$$\sum_k \rho(A_k X_j A_k) \qquad\qquad \text{with } X_j \equiv \sum_i B_i C_j B_i. \tag{5}$$

The fact that the A_ks don't commute with the B_is implies that (5) is typically not equal to $\rho(X_j)$, which would be the expectation value of C_j if the B-measurement occurs but the A-measurement does not.

If this application of the artificial approach were valid, then the inequality of (5) and $\rho(X_j)$ would say that the occurrence of the A-measurement could affect the outcome statistics of the C-measurement, even though A and C are causally disjoint. This would imply that faster-than-light communication is possible in relativistic QFT, which is why it's dubbed a *paradox*, but section 9 will explain why that conclusion is too hasty.

[14]Sorkin (1993); reviewed in Papageorgiou and Fraser (2024), section 2.2

[15]Section 2

9 Sorkin's paradox in the natural approach

Now consider what happens to Sorkin's paradox when we use what section 5 called the *natural approach.* Define

$$\Omega_i \equiv C_j B_i$$

so that the quantity (5) may be written

$$\sum_{k,i} \rho(A_k \Omega_i^\dagger \Omega_i A_k). \tag{6}$$

The natural approach uses a model in which measurements occur as physical processes within the model itself. If ρ is a state in which the A-measurement occurs, then equation (4) holds whenever Ω is a product of projection operators representing a sequence of outcomes of measurements localized in spacetime regions outside the causal past of the region where the A-measurement occurs. This implies

$$\sum_{k,i} \rho(A_k \Omega_i^\dagger \Omega_i A_k) = \sum_i \rho(\Omega_i^\dagger \Omega_i) + \epsilon = \sum_i \rho(B_i C_j B_i) + \epsilon, \tag{7}$$

which says that any effect of the occurrence of the A-measurement on the outcome statistics of the C-measurement is utterly negligible. This shows that using Sorkin's setup to achieve faster-than-light communication would be as difficult as observing quantum interference between the two different microstates of the macroscopic environment associated with two different outcomes of a sharp measurement. For all practical purposes, it's impossible.

10 Does the natural approach resolve the paradox?

Section 9 showed that Sorkin's paradox in its original form is too simplistic. The paradox arises when we use what sections 4-6 called the *artificial approach* to handling measurements in quantum theory, and section 9 showed that using the *natural approach* changes the conclusion. The natural approach accounts for the fact that measurement is a physical process. The selective state-update rule is still needed to account for specific outcomes, but we should not apply either of the state-update rules arbitrarily. We are only allowed to apply them after a measurement has occurred. The natural approach uses a model that can – in principle – tell us when measurements occur. When applied to Sorkin's setup, it tells us that if the A-measurement does occur, then a B-measurement of the type that would lead to Sorkin's paradox cannot occur.

Before we declare that Sorkin's paradox has been resolved, though, we should remember that the *natural approach* currently relies on intuition about the physics of measurement that has not been explicitly checked in relatively natural models like quantum electrodynamics or quantum chromodynamics.[16] In particular, it relies on the assumption about self-consistency that was highlighted in section 3. Section 9 "resolves" Sorkin's paradox only in the sense that it subsumes the paradox into a more fundamental question, one that already underlies all applications of quantum theory.

[16]Determining which states represent a *single particle* is already very difficult in these models, nevermind analyzing the dynamics of states involving measurement equipment made of jillions of particles.

11 Some research

This article's main message is that Sorkin's paradox doesn't treat measurement the way it should be treated in principle – as a physical process that requires physical resources. According to section 1 in Mandrysch and Navascues (2024), "The accepted resolution of Sorkin's paradox is to acknowledge that the set of quantum operations which can be conducted within a finite region of space-time is certainly smaller than previously envisaged." The intuition in section 9 is consistent with this statement, but it's just intuition. This section highlights two compromises that have been used to enable a more substantial analysis.

Jubb (2022) uses what sections 4-6 call the *artificial approach*, but instead of assuming that all so-called observables are measurable (which would lead to Sorkin's paradox), the author asks which observables must be excluded as unmeasurable to avoid the paradox.[17] As expected, part of the conclusion is that the paradox can be avoided by limiting the set of observables that are assumed to be measurable.

The results reported in Bostelmann *et al* (2021)[18] use an approach that is often used in studies of decoherence: measurements are implemented by modifying the system's dynamics in a time-dependent way so that the interaction between the thing being measured and the rest of the system (the *probe*) is only active during a brief time interval, representing the time at which the measurement occurs. This falls short of what section 5 calls the *natural approach* because the time interval in which the measurement occurs is controlled by artificially modifying the dynamics,[19] but it at least tries to account for the physical process of measurement.[20] Part of the conclusion is that the type of temporary interaction used by the authors automatically limits the set of measurable observables in a way that again avoids Sorkin's paradox.

[17]Beckman *et al* (2002) applies a similar criterion to Wilson loop observables in Yang-Mills theory.

[18]These results use an approach developed in Fewster and Verch (2020) and reviewed in Fewster (2019).

[19]Section 2 in Fewster (2019) acknowledges this ("the interactions of nature are not ours to change").

[20]Section 2 in Fewster (2019) acknowledges this, too ("the couplings [with the probe] represent a proxy for an experimental design that engineers interactions to occur in the apparatus and tries to screen out extraneous influences").

12 References

(Open-access items include links.)

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13 References in this series

Article **03431** (https://cphysics.org/article/03431):
"The Core Principles of Quantum Theory and the Nature of Measurement"

Article **21916** (https://cphysics.org/article/21916):
"Local Observables in Quantum Field Theory"

Article **48968** (https://cphysics.org/article/48968):
"The Geometry of Spacetime"